October Math Gems

PROBLEM OF THE WEEK 13

§1 Problems

Problem 1.1. If the line $kx + 4y = 6$ passes through the point of intersection of the two lines $2x + 3y = 4$ and $3x + 4y = 5$

Problem 1.2. If p_1 and p_2 are length of the perpendicular from the origin on the two lines given by $x \sec A + y \csc A = m$ and $x \cos A + y \sin A = m \cos 2A$ then $4(p_1)^2 + (p_2)^2 = ...$ with respect to m

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Problem 1.3. If (4-,5) is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of the other diagonal is....

Problem 1.4. A triangle has vertices $A(0, b); B(0, 0); C(a, 0)$. If its median AD and BE are mutually perpendicular then $a = ...$ with respect to b

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Problem 1.5. Points (3,3),(h,0),(0,k) are collinear and $\frac{a}{h} + \frac{b}{k} = \frac{1}{3}$. Then what is the value of a, b

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Problem 1.6. Let $p(x)$ be a cubic polynomial with zeros α, β, γ. If $\frac{p(\frac{1}{2})+p(\frac{-1}{2})}{p(0)} = 100$ find $\sqrt{\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}}$

Problem 1.7. Find the value of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$

Problem 1.8. Consider the equation $x^3 - (1+\cos\theta+\sin\theta)x^2 + (\cos\theta\sin\theta+\cos\theta+\sin\theta)x - \sin\theta\cos gq = 0$ The roots of equation are x_1, x_2, x_3 Find the value of $(x_1)^2 + (x_2)^2 + (x_3)^2$

Problem 1.9. The inverse function of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} =$

Problem 1.10. Let $f(x) = \frac{ax+b}{cx+d}$ then $fof(x) = x$ Find d

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Problem 1.11. Let $f(x) = \frac{ax}{x+1}$ for x not equal -1. Then what is the value of a if $f(f(x)) = x$

Problem 1.12. The function f is one-to-one and the sum of all the intercepts of the graph is 5. The sum of all intercepts of the graph $y = f^{-1}(x)$ is

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Problem 1.13. $\frac{f(x)}{g(x)} = x - 2$ with remainder $4 - 2x$ Find $g(x)$ if $f(x) = x^3 - 3x^2 + x + 2$

Problem 1.14. The expansion $\frac{1}{\sqrt{4x+1}}((\frac{1+\sqrt{4x+1}}{2})^2 - (\frac{1-\sqrt{4x-1}}{2})^2)$ is a polynomial of x degree $=$

Problem 1.15. b, a are zeros of $h(x) = 3x^2 - 6x + 12$ find the value of $a^{-1} + b^{-1}$

Problem 1.16. If α, β, γ are zeros of polynomial $x^3 - x - 1$, then the value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$

Problem 1.17. suppose $f(x) = ax + b$ and $g(x) = bx + a$ where a, b are positive integers. If $f(g(50)) - g(f(50)) = 28$ then the product of ab cam have the value of

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Problem 1.18. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$ where p, q are real number, then $p + q =$

Problem 1.19. The sum of n terms of two arithmetic series are in the ratio of $2n + 3 : 6n + 5$ then the ratio for their $13^{th} termsis$

Problem 1.20. The p^{th} term of an arithmetic progression is q and the q^{th} term is p the the 10^{th} term= (with respect to p, q)
